

Topics : Vector, Solution of Triangle, Function

Type of Questions	M.M., Min.
Single choice Objective (no negative marking) Q.1	(3 marks, 3 min.) [3, 3]
Multiple choice objective (no negative marking) Q.2, 3	(5 marks, 4 min.) [10, 8]
Subjective Questions (no negative marking) Q. 4, 5, 6, 7, 8	(4 marks, 5 min.) [20, 25]

1. P, Q have position vectors \vec{a} & \vec{b} relative to the origin 'O' & X, Y divide \vec{PQ} internally and externally respectively in the ratio 2 : 1 . Vector $\vec{XY} =$

- (A) $\frac{3}{2}(\vec{b} - \vec{a})$ (B) $\frac{4}{3}(\vec{a} - \vec{b})$ (C) $\frac{5}{6}(\vec{b} - \vec{a})$ (D) $\frac{4}{3}(\vec{b} - \vec{a})$

2. If in a triangle ABC, $b \cos^2 \frac{A}{2} + a \cos^2 \frac{B}{2} = \frac{3c}{2}$, then

- (A) $c^2 \geq ab$ (B) $\frac{a}{c} + \frac{c}{b} + \frac{b}{a} \geq 3$ (C) $\frac{a+c}{2c-a} + \frac{b+c}{2c-b} \geq 4$ (D) a, b, c are in A.P.

3. If 'O' is the circumcentre of the ΔABC and R_1, R_2 and R_3 are the radii of the circumcircles of triangles OBC, OCA & OAB respectively, then $\frac{a}{R_1} + \frac{b}{R_2} + \frac{c}{R_3}$ has the value equal to

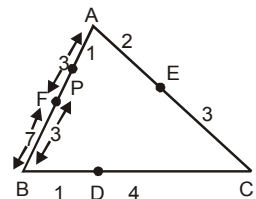
- (A) $\frac{abc}{2R^3}$ (B) $\frac{R^3}{abc}$ (C) $\frac{4\Delta}{R^2}$ (D) $\frac{abc}{R^3}$

4. In a ΔABC , prove that $\frac{(a+b+c)^2}{a^2+b^2+c^2} = \frac{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}}{\cot A + \cot B + \cot C}$

5. If the solution of the equation $\sqrt{2+\sqrt{2+\sqrt{2+x}}} + \sqrt{3}\sqrt{2-\sqrt{2+\sqrt{2+x}}} = 2x$, where $x > 0$ is given by $x = a \cos(b\pi/c)$ where argument of cosine function lies in $[0, \pi/2)$, a, b, c $\in \mathbb{N}$ and b, c are relatively prime, find the value of $(a + b + c)$.

6. ABCD is a quadrilateral and E the point of intersection of the lines joining the middle points of opposite sides. Show that the resultant of $\vec{OA}, \vec{OB}, \vec{OC}$ and \vec{OD} is equal to $4 \vec{OE}$, where O is any point.

7. The point D, E, F divide the respective sides of ΔABC in the ratio as shown in figure. P is a point in AB which divides AB in the ratio 1 : 3 internally prove that $5(\vec{AD} + \vec{BE} + \vec{CF}) = 2\vec{CP}$



8. Find all the values of a for which the function $f(x) = (a^2 - 3a + 2) \cos\left(\frac{x}{2}\right) + (a - 1)x$ possesses critical points.

Answers Key

1. (D) 2. (A)(B)(C) 3. (C)(D) 5. 37
8. $(-\infty, 0] \cup [4, \infty) \cup \{1\}$

